UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education
Advanced Subsidiary Level and Advanced Level

5299987921

MATHEMATICS **9709/13**

Paper 1 Pure Mathematics 1 **(P1)** **May/June 2011**

1 hour 45 minutes

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF9)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
The use of an electronic calculator is expected, where appropriate.
You are reminded of the need for clear presentation in your answers.

At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.
The total number of marks for this paper is 75.
Questions carrying smaller numbers of marks are printed earlier in the paper, and questions carrying larger numbers of marks later in the paper.

This document consists of **3** printed pages and **1** blank page.

1 The coefficient of x^3 in the expansion of $(a + x)^5 + (1 - 2x)^6$, where a is positive, is 90. F
of a.

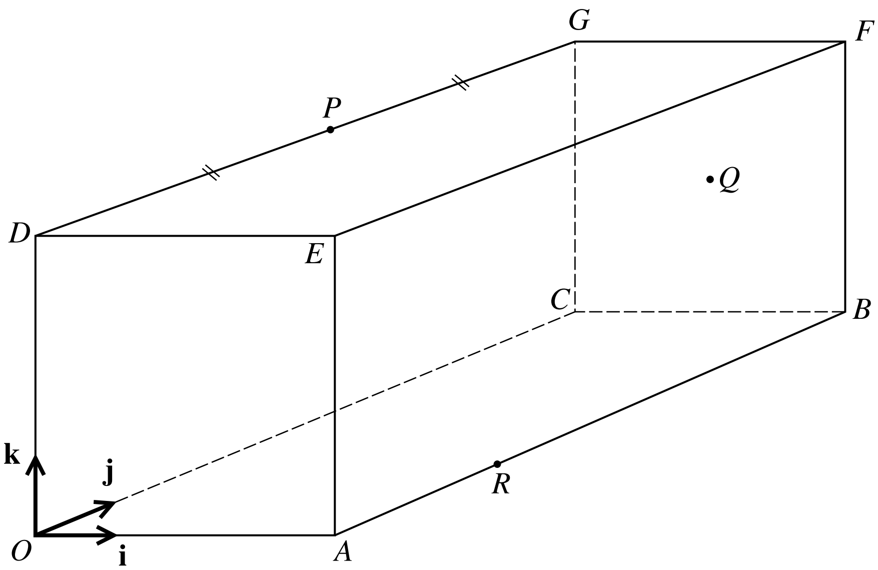
2 Find the set of values of m for which the line $y = mx + 4$ intersects the curve $y = 3x^2 - 4x + 7$ at tw
distinct points. [5]

3 The line $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are positive constants, meets the x-axis at P and the y-axis at Q. Given that $PQ = \sqrt{(45)}$ and that the gradient of the line PQ is $-\frac{1}{2}$, find the values of a and b. [5]

4 **(a)** Differentiate $\frac{2x^3 + 5}{x}$ with respect to x. [3]

(b) Find $\int (3x - 2)^5 \, dx$ and hence find the value of $\int_0^1 (3x - 2)^5 \, dx$. [4]

5



In the diagram, $OABCDEFG$ is a rectangular block in which $OA = OD = 6$ cm and $AB = 12$ cm. The unit vectors **i**, **j** and **k** are parallel to $\overrightarrow{OA}$, $\overrightarrow{OC}$ and $\overrightarrow{OD}$ respectively. The point P is the mid-point of DG, Q is the centre of the square face $CBFG$ and R lies on AB such that $AR = 4$ cm.

(i) Express each of the vectors $\overrightarrow{PQ}$ and $\overrightarrow{RQ}$ in terms of **i**, **j** and **k**. [3]

(ii) Use a scalar product to find angle RQP. [4]

6 **(a)** A geometric progression has a third term of 20 and a sum to infinity which is three times the first term. Find the first term. [4]

(b) An arithmetic progression is such that the eighth term is three times the third term. Show that the sum of the first eight terms is four times the sum of the first four terms. [4]

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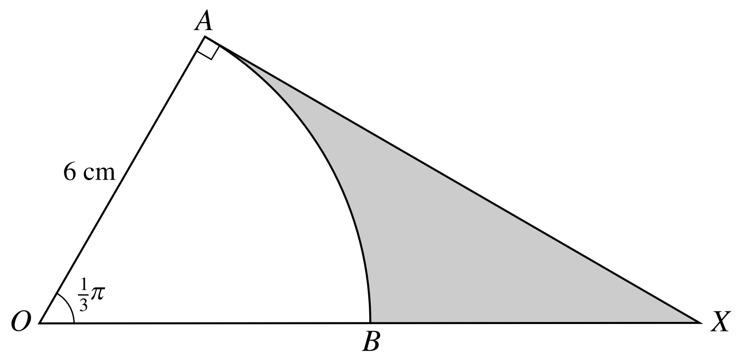

In the diagram, AB is an arc of a circle, centre O and radius 6 cm, and angle $AOB = \frac{1}{3}\pi$ radians. The line AX is a tangent to the circle at A, and OBX is a straight line.

(i) Show that the exact length of AX is $6\sqrt{3}$ cm. [1]

Find, in terms of π and $\sqrt{3}$,

(ii) the area of the shaded region, [3]

(iii) the perimeter of the shaded region. [4]

8 **(i)** Prove the identity $\left(\dfrac{1}{\sin\theta} - \dfrac{1}{\tan\theta}\right)^2 \equiv \dfrac{1-\cos\theta}{1+\cos\theta}$. [3]

(ii) Hence solve the equation $\left(\dfrac{1}{\sin\theta} - \dfrac{1}{\tan\theta}\right)^2 = \frac{2}{5}$, for $0° \leqslant \theta \leqslant 360°$. [4]

9 A curve is such that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{2}{\sqrt{x}} - 1$ and $P\,(9, 5)$ is a point on the curve.

(i) Find the equation of the curve. [4]

(ii) Find the coordinates of the stationary point on the curve. [3]

(iii) Find an expression for $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ and determine the nature of the stationary point. [2]

(iv) The normal to the curve at P makes an angle of $\tan^{-1} k$ with the positive x-axis. Find the value of k. [2]

10 Functions f and g are defined by

$$\mathrm{f} : x \mapsto 3x - 4, \quad x \in \mathbb{R},$$
$$\mathrm{g} : x \mapsto 2(x-1)^3 + 8, \quad x > 1.$$

(i) Evaluate fg(2). [2]

(ii) Sketch in a single diagram the graphs of $y = \mathrm{f}(x)$ and $y = \mathrm{f}^{-1}(x)$, making clear the relationship between the graphs. [3]

(iii) Obtain an expression for $\mathrm{g}'(x)$ and use your answer to explain why g has an inverse. [3]

(iv) Express each of $\mathrm{f}^{-1}(x)$ and $\mathrm{g}^{-1}(x)$ in terms of x. [4]

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